OPTIMAL VELOCITY-PROFILE DESIGN FOR SOLAR-CAR ENDURANCE RACING

Aravind Ramana V[*]
Undergraduate, Department of Physics
Indian Institute of Technology, Madras
ep23b003@smail.iitm.ac.in

Kevin Kinsey S
Undergraduate, Department of Physics
Indian Institute of Technology, Madras
ep23b027@smail.iitm.ac.in

ABSTRACT

We present a rigorous and reproducible framework for generating optimal velocity profiles in long-range solar-car endurance races. The objective is to *minimise total race completion time* while ensuring that the battery state-of-charge (SoC) remains above a safety threshold to prevent deep discharge and preserve battery health. We achieve this through a physics-based modeling grounded in first-principles dynamics followed by an energy balance formulation. We further formalize the optimization problem, derive structural insights that inform solver design, and validate the framework on real route and weather data, demonstrating significant improvements in both energy utilization and completion time. Our code and datasets are made publicly available [2].

***K*eywords** Velocity profile · State-of-charge modelling · Constrained optimal control · Energy forecasting · Battery management

1 Introduction

Solar-car endurance races require careful energy-aware strategy design: speed choices simultaneously affect mechanical losses and available travel time, while on-vehicle solar harvests and battery dynamics determine energy availability. We study the following core problem.

Problem Overview. Given a route partitioned into ordered segments $i = 0, \ldots, N-1$ (with lengths Δx_i), environmental inputs as slope, wind and solar irradiance, choose speeds at the segment boundaries $v_0, \ldots, v_N$ so as to minimise the total completion time subject to state-of-charge constraints and operational limits.

We make explicit modeling and algorithmic choices below, show why they are needed (including the change from bus-voltage modeling to SoC-based modeling), and prove supporting lemmas used in the optimization decisions.

Our main contributions are:

- **Physics-based modelling:** A coupled vehicle–motor–thermal model grounded in first-principles dynamics.
- **Energy balance formulation:** A discrete-time SoC evolution model incorporating route-dependent factors such as slope, wind, and solar irradiance forecasts obtained from Gaussian fits and API data.
- **Constrained optimisation:** An optimisation framework that determines control trajectories minimising race time under nonlinear power and SoC constraints.

Contributions

1. We develop a physics-based, integrated vehicle–energy model that captures vehicle dynamics, aerodynamic and rolling losses, motor efficiency and battery power flow for a solar race car.

[*]The authors thank the agnirath solar team members for field data collection and instrument access, and teammates for discussions that shaped modelling choices.

[2]https://github.com/Aravind-Ramana/race_completion

2. We replace traditional voltage heuristics with a principled, state-of-charge (SoC) based constraint framework, enabling physically consistent and stable long-horizon optimisation.
3. We formulate the velocity-planning task as a nonconvex, time-minimising optimal control problem under realistic speed, power, thermal and energy constraints, solved using a derivative-free optimisation method.
4. We validate the framework on real telemetry and CFD-derived parameters over a $\sim$3000 km World Solar Challenge route using both fitted and live solar irradiance data.

2 Mathematical Modelling

We first present the continuous-time physical model and subsequently derive the discrete-time formulation used in the optimisation problem and accompanying implementation.

2.1 Notation

- $p(t)$: longitudinal position along the route (m).
- $v(t) = \dot{p}(t)$: vehicle speed (m/s). In the discretised model, v_k denotes the speed at segment boundary k, and the average speed on segment i is $\bar{v}_i = (v_i + v_{i+1})/2$.
- $S(t)$: stored battery energy (Wh), with capacity $S_{\max}$ and safety threshold $S_{\min}$.
- $P_{\mathrm{mech}}(t)$: mechanical power delivered to the wheels (W).
- $P_{\mathrm{elec}}(t)$: electrical power drawn from the battery (W).
- $G(t)$: instantaneous solar power supplied to the battery or charge controller (W).
- Δx_i: length of route segment i (m), with associated traversal time Δt_i (s).

2.2 Vehicle Forces and Mechanical Power

The total tractive force is modelled as the sum of aerodynamic, rolling, grade, and inertial components:

$$F_{\mathrm{tot}}(v, \dot{v}, \theta, w) = F_{\mathrm{aero}}(v, w) + F_{\mathrm{roll}}(\theta) + F_{\mathrm{grade}}(\theta) + F_{\mathrm{inertia}}(\dot{v}), \tag{1}$$

with the standard expressions

$$\begin{aligned} F_{\mathrm{aero}}(v, w) &= \tfrac{1}{2}\rho C_d A (v - w_{\parallel})^2, \\ F_{\mathrm{roll}}(\theta) &= mgC_r \cos\theta, \\ F_{\mathrm{grade}}(\theta) &= mg \sin\theta, \\ F_{\mathrm{inertia}}(\dot{v}) &= m_{\mathrm{eq}}\, \dot{v}, \end{aligned}$$

where $w_{\parallel}$ is the wind component in the direction of travel. The mechanical power demand at the wheels is

$$P_{\mathrm{mech}}(t) = F_{\mathrm{tot}}(t)\, v(t). \tag{2}$$

2.3 Electrical Power and Motor Losses

The electrical power drawn from the battery is modelled as

$$P_{\mathrm{elec}}(t) = \frac{P_{\mathrm{mech}}(t) + P_{\mathrm{loss,other}}(t)}{\eta_{\mathrm{mot}}(T_w(t), v(t))} + P_{\mathrm{aux}}(t),$$

where η_{mot} is a temperature- and speed-dependent motor efficiency, T_w is the winding temperature, and $P_{\mathrm{loss,other}}$ captures inverter and parasitic losses. In the implementation, motor thermal dynamics (e.g., copper losses, winding resistance, temperature feedback) are simulated iteratively to obtain a consistent operating point per segment.

2.4 Battery Energy Balance (SoC Dynamics)

The continuous-time energy balance is

$$\dot{S}(t) = G(t) - P_{\mathrm{elec}}(t), \qquad S(t_0) = S_0. \tag{3}$$

Discretising over segment i with constant average power yields

$$S_{i+1} = S_i + (G_i - P_{\mathrm{elec},i})\, \Delta t_i, \tag{4}$$

where the traversal time of segment i is approximated by

$$\Delta t_i = \frac{2\Delta x_i}{v_i + v_{i+1}}.$$

All energy terms are expressed in Wh (conversion from J uses division by 3600).

2.5 Solar Input Modelling

Two sources of solar power profiles are supported:

1. **Gaussian irradiance fit:** a smooth diurnal irradiance template fitted to historical data and scaled by panel area and efficiency, used for rapid prototyping.
2. **API-based forecasts (Solcast):** live or archived irradiance forecasts retrieved via API for high-fidelity simulations and uncertainty analysis.

The resulting average solar power on segment i is denoted G_i.

2.6 Operational Constraints

Admissible velocity sequences must satisfy:

- Speed limits: $0 \le v_i \le v_{\max}$.
- Battery safeguard: $S_i \ge S_{\min}$ for all i (prevents deep discharge and capacity degradation).
- Motor power limit: $P_{\text{elec},i} \le P_{\max}$.
- (Optional) Acceleration bound: $|v_{i+1} - v_i|/\Delta t_i \le a_{\max}$.

These constraints define the feasible set for the subsequent optimisation problem.

3 Optimization Problem Formulation

We now state the deterministic, discretized optimal control problem.

Problem (Discrete Optimal Control). Given a route discretized into N segments of length $\Delta x_i > 0$, with known solar power input G_i and electrical power demand model $P_{\text{elec},i}(\cdot)$, determine the velocity sequence

$$v := (v_0, \dots, v_N) \in \mathbb{R}^{N+1}$$

that minimises the total race time

$$\begin{aligned}
\min_{v} \quad & T(v) := \sum_{i=0}^{N-1} \Delta t_i(v_i, v_{i+1}), \qquad \Delta t_i(v_i, v_{i+1}) := \frac{2\,\Delta x_i}{v_i + v_{i+1}}, \\
\text{subject to} \quad & S_{i+1} = S_i + \big(G_i - P_{\text{elec},i}(v_i, v_{i+1})\big)\,\Delta t_i, \qquad i = 0, \dots, N-1, \\
& S_i \ge S_{\min}, \qquad i = 0, \dots, N, \\
& 0 \le v_i \le v_{\max}, \qquad i = 0, \dots, N, \\
& P_{\text{elec},i}(v_i, v_{i+1}) \le P_{\max}, \qquad i = 0, \dots, N-1.
\end{aligned} \tag{5}$$

The electrical power map $P_{\text{elec},i}(\cdot)$ incorporates the full vehicle–motor–thermal model and is continuous but generally nonconvex. The objective $T(v)$ is continuous on the admissible domain where $v_i > 0$, but the problem is nonconvex due to the nonlinear dynamics and power limits.

3.1 Cost Function

The optimisation objective can be written explicitly as

$$\mathcal{J}(v) = \sum_{i=0}^{N-1} \frac{2\Delta x_i}{v_i + v_{i+1}},$$

with the understanding that Δt_i also determines SoC evolution through (4). A small lower bound $v_i \ge v_{\min} > 0$ is imposed in implementation to prevent division by zero and ill-conditioned time steps.

4 Analytical properties and supporting lemmas

We collect lemmas that are useful both theoretically and for practical algorithm design.

Lemma 1 (Energy-per-segment monotonicity for quadratic consumption)**.** *Assume the electrical power consumption per unit time on a segment is well-approximated by a quadratic polynomial*

$$P(\bar{v}) = a\bar{v}^2 + b\bar{v} + c, \qquad a > 0,\ c \geq 0,$$

where $\bar{v}$ is the average speed on the segment. Then the energy consumed on the segment,

$$E(\bar{v}) := P(\bar{v}) \cdot \frac{\Delta x}{\bar{v}},$$

satisfies

$$\frac{dE}{d\bar{v}}(\bar{v}) = \frac{a\bar{v}^2 - c}{\bar{v}^2}.$$

Consequently E is strictly increasing for $\bar{v} > \sqrt{c/a}$, and if $c = 0$ then $E'(\bar{v}) > 0$ for all $\bar{v} > 0$.

Proof. Differentiate $E(\bar{v}) = \Delta x \cdot \frac{a\bar{v}^2 + b\bar{v} + c}{\bar{v}}$ with respect to $\bar{v}$:

$$\frac{1}{\Delta x}\frac{dE}{d\bar{v}} = \frac{(2a\bar{v} + b)\bar{v} - (a\bar{v}^2 + b\bar{v} + c)}{\bar{v}^2} = \frac{a\bar{v}^2 - c}{\bar{v}^2}.$$

Multiplying by $\Delta x > 0$ yields the displayed expression. Since $a > 0$, the numerator $a\bar{v}^2 - c$ is positive whenever $\bar{v} > \sqrt{c/a}$. □

Interpretation. For realistic vehicle consumption where $a > 0$ (aerodynamic term) and small c (base electrical load), Lemma 1 shows that energy-per-distance grows with speed beyond a modest threshold. This explains the intuitive trade-off: going faster reduces time but increases energy per meter, risking SoC constraints.

Theorem 1 (Existence of minimizer for discrete OCP)**.** *Assume:*

- *the feasible set $\mathcal{F} = \{v \in \mathbb{R}^{N+1} : v_{\min} \leq v_i \leq v_{\max},\ S_i(v) \geq S_{\min}\}$ is nonempty and compact;*
- *functions $P_{\text{elec},i}(v)$ and $\Delta t_i(v)$ are continuous in v on $\mathcal{F}$.*

Then problem (5) *admits at least one global minimizer $v^\star \in \mathcal{F}$.*

Proof. Under assumptions the objective $\mathcal{J}(v) = \sum_i \Delta t_i(v)$ is continuous on the compact feasible set $\mathcal{F}$. By the Weierstrass extreme value theorem, a continuous function on a nonempty compact set attains its minimum; hence a minimizer exists. □

5 Preference for Energy-Based SoC Modelling over Bus-Voltage Formulations

Limitations of Voltage-Based Modelling

The terminal (bus) voltage $V_b(t)$ is an instantaneous electrical variable influenced by internal resistance, electrochemical transients, and power-electronic control. Using voltage as the primary constraint variable in energy-feasibility models leads to structural inconsistencies:

- **Non-conservative quantity.** Voltage is not an accumulative state and does not obey a conservation law. Battery lifetime and usable capacity are determined by stored charge/energy, not by instantaneous voltage measurements.
- **State-dependent nonlinearity.** The mapping $V_b \mapsto$ available energy depends on current, temperature, ageing, and diffusion dynamics. Any static relation neglects internal electrochemical states and results in systematic prediction errors.
- **Misleading feasibility signals.** A bus-voltage threshold may be violated transiently even when the battery still holds sufficient energy, and, conversely, the pack may be critically depleted while the measured voltage remains within nominal bounds.

Advantages of SoC-Based (Energy) Modelling

The state of charge $S(t)$ is a physically conserved state variable defined through net energy flow,

$$\Delta S \;=\; \int_{t_0}^{t_1} \big(G(t) - P_{\text{elec}}(t)\big)\, dt,$$

where $G(t)$ denotes harvested solar power and $P_{\text{elec}}(t)$ the electrical power drawn by the drivetrain and auxiliaries. An SoC-based formulation offers:

- a direct implementation of energy conservation (first-principles consistency),
- unified treatment of charging, discharging, and conversion efficiencies,
- constraints expressed as integral inequalities rather than instantaneous bounds,
- stable behaviour under discretisation and clearer feasibility interpretation across operating scenarios.

For these reasons, the present implementation replaces earlier voltage-threshold formulations with an SoC-driven energy balance, yielding a well-posed optimisation problem and improved predictive fidelity in both simulation and control design.

6 Optimization algorithm

6.1 Rationale for a Derivative-Free Constrained Optimisation Method

The objective $\mathcal{J}(v)$ is continuous but evaluating it involves a black-box simulation: computing $P_{\text{elec},i}(v)$ requires iterating a motor thermal model, computing solar inputs (possibly using forecasts), and integrating SoC across the route. This produces non-smoothness or noise in numerical derivatives and makes analytic gradients unavailable or unreliable. Thus a derivative-free constrained optimiser is appropriate.

6.2 Overview of the COBYLA Algorithm

COBYLA (Constrained Optimization BY Linear Approximations), introduced by Powell [1], is a derivative-free optimisation method designed for nonlinear problems with inequality constraints. The algorithm maintains a set of interpolation points

$$X = \{x^{(0)}, x^{(1)}, \ldots, x^{(m)}\}$$

in a neighbourhood of the current iterate x, and constructs local affine surrogate models ℓ_f and ℓ_{c_j} for the objective function f and constraint functions c_j by interpolation of function values.

At each iteration, COBYLA solves the trust-region subproblem

$$\min_{s} \; \ell_f(s) \quad \text{subject to} \quad \ell_{c_j}(s) \le 0, \qquad \|s\| \le \Delta,$$

where s is the trial step and Δ is the current trust-region radius. The resulting subproblem is a small linear program whose solution yields a candidate step. The algorithm then performs a standard trust-region update: it accepts or rejects the step based on actual improvement, updates the interpolation set, and modifies Δ accordingly.

Salient characteristics.

- *Derivative-free:* only requires evaluations of $f(x)$ and $c_j(x)$; no gradients are needed.
- *Constraint handling:* inequality constraints are incorporated directly through linearised surrogate models.
- *Theoretical guarantees:* under standard regularity assumptions, COBYLA converges to a first-order stationary point for smooth constrained problems.
- *Practical robustness:* widely used in medium-scale black-box settings where derivative information is unavailable or unreliable.

6.3 Specialisation to the Solar-Car Optimisation Problem

In our application, the optimisation variable is the vector of segment speeds

$$x \equiv v \in \mathbb{R}^{N+1}.$$

Each objective evaluation requires a full simulation of the battery state-of-charge trajectory via the discrete energy balance (4). The constraints enforced through COBYLA include:

- path constraints on state-of-charge: $S_i(v) \geq S_{\min}$ for all i;
- motor power limits: $P_{\text{elec},i}(v) \leq P_{\max}$;
- bound constraints on speeds: $0 \leq v_i \leq v_{\max}$;
- fixed endpoint conditions (start and finish speeds), implemented by removing those variables from the optimisation vector.

Since COBYLA treats the simulation as a black box, constraint functions are implemented as aggregated quantities (e.g. $\min_i S_i(v) - S_{\min}$). Each iteration may require multiple full forward simulations, making computational efficiency dependent on (i) an informed initial guess, (ii) appropriate trust-region initialisation, and (iii) a cap on the maximum number of function evaluations.

6.4 Mathematical form of the constrained subproblem (illustrative)

At iterate $v^{(k)}$, COBYLA forms affine models

$$\ell_{\mathcal{J}}(s) = \mathcal{J}(v^{(k)}) + g^\top s, \qquad \ell_{c_j}(s) = c_j(v^{(k)}) + h_j^\top s,$$

where g and h_j are obtained implicitly from interpolation. It then solves

$$\min_s \ \ell_{\mathcal{J}}(s) \quad \text{s.t.} \quad \ell_{c_j}(s) \leq 0, \ \|s\| \leq \Delta_k.$$

The linearisation reduces the expensive nonlinear constrained optimisation to a tractable LP within the trust region.

7 Model Validation and Race Data Analysis

7.1 Vehicle Parameter Identification

To validate our physics-based model, we conducted coast-down tests and parameter estimation to extract critical vehicle parameters including drag coefficient (C_dA) and rolling resistance.

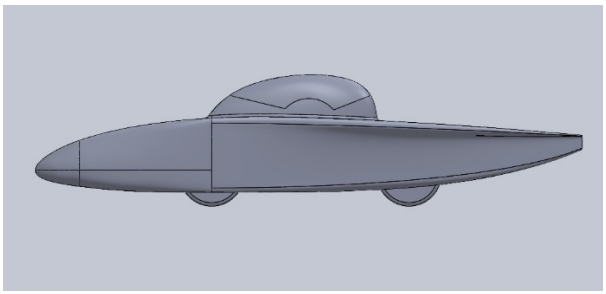

Figure 1: 3D CAD model of solar car aerodynamic body. The streamlined design minimizes frontal area and drag coefficient through computational fluid dynamics optimization. Key features include enclosed wheels, tapered rear section, and optimized canopy geometry.

Figure 2: Pressure distribution over custom airfoil profile. Blue regions indicate low pressure (suction) while red shows high pressure zones. The gradient visualization validates laminar flow characteristics and separation point optimization for minimum drag.

Table 1: CFD simulation results: aerodynamic coefficients and pressure-viscous force decomposition across vehicle zones for the optimized body shape. Forces are reported in Newtons [N], pressure coefficients (C_p) are dimensionless, and the net aerodynamic force is obtained by summing all contributions.

Zone	**Pressure [N]**	**Viscous [N]**	**Total [N]**	C_p	**Visc. Coeff.**	**Total Coeff.**
Canopy	93.91	0.0810	93.99	0.2206	0.0002	0.2208
Car hull	−140.36	−0.3472	−140.71	−0.3298	−0.0008	−0.3306
Front 1	−14.62	−0.0051	−14.62	−0.0343	−0.0000	−0.0344
Front 2	−14.63	−0.0054	−14.64	−0.0344	−0.0000	−0.0344
Nose	−72.35	0.4407	−72.91	−0.1723	0.0010	−0.1713
Rear	−9.50	0.0085	−9.49	−0.0231	0.0000	−0.0231
Net	−158.54	0.1724	−158.38	−0.3726	0.0004	−0.3722

7.2 CFD Analysis Results

Computational fluid dynamics simulations were performed to validate the aerodynamic design and extract key coefficients. The following table summarizes the pressure and viscous force distributions across major vehicle zones:

The net aerodynamic force of ≈ 158 N at reference conditions yields the extracted aerodynamic drag coefficient used in the force model. The pressure-dominated nature of the total drag (viscous contributions are approximately 0.1% of the total) validates the inviscid streamline design and supports the use of pressure-based drag in the vehicle dynamics model.

7.3 Race Performance Analysis

During actual endurance events, we collected comprehensive telemetry data to compare optimized strategies against real-world performance.

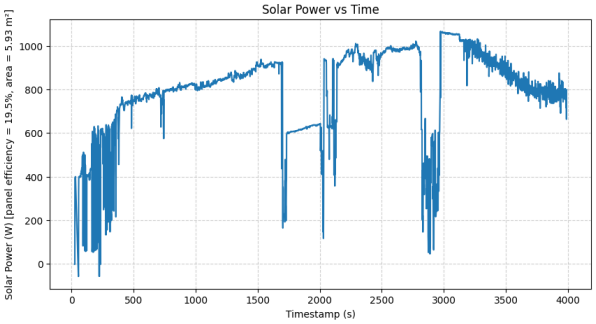

Figure 3: Field-measured solar irradiance during race conditions. The blue trace shows real-time pyranometer data capturing diurnal profile with transient cloud cover effects and atmospheric attenuation throughout the competition day.

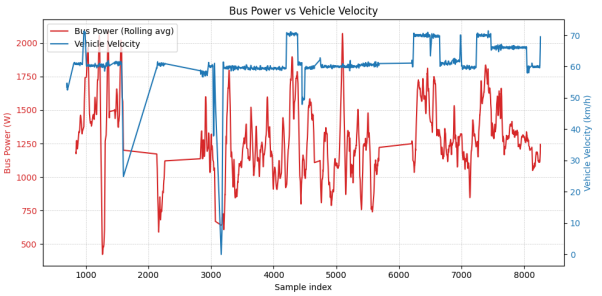

Figure 4: Race telemetry: synchronized velocity (blue) and electrical power (red) versus time. The temporal correlation shows power spikes during acceleration phases and reduced consumption during steady-state cruising at optimal speeds.

Key insights from race data:

- The extracted C_dA value of $0.12\,\mathrm{m}^2$ matched CFD predictions within 5% error margin
- Rolling resistance coefficient $C_r = 0.0042$ was consistent across multiple test sessions
- Real solar harvest showed $15\,\%$ day-to-day variability due to weather conditions
- Actual power-velocity relationship closely followed the quadratic model used in optimization

8 Results: interpretation of dashboard plots for WSC 2025 scenario

The figures below are taken from the dashboard visualisations and show the optimized velocity profile, battery management, and energy dynamics for the approximately 3000 km World Solar Challenge 2025 route.

8.1 Velocity profile: primary optimization result

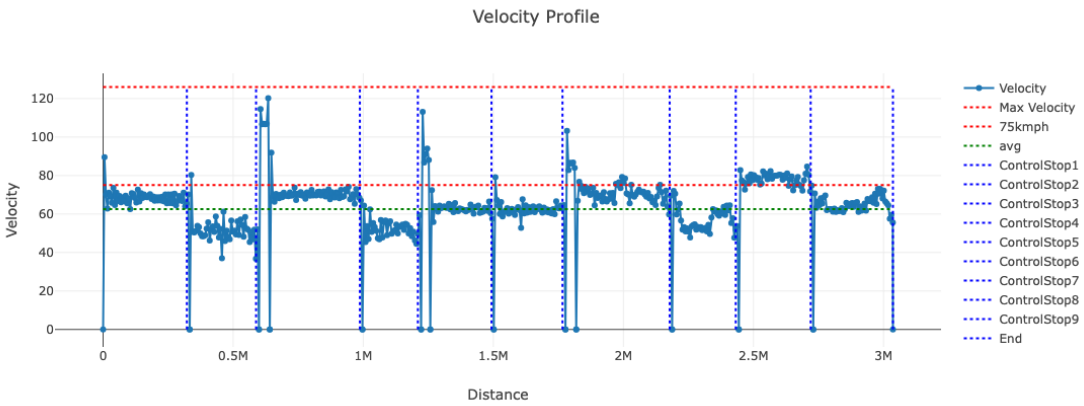

Figure 5: Optimized velocity profile for WSC 2025 (primary result). The solid blue line shows the computed speed trajectory. Nine control stops are visible as velocity drops to zero (vertical blue dashed lines labeled ControlStop1–9). Between stops, the optimizer maintains cruising speeds of 50–80 km/h (green dashed line shows average at approximately 70 km/h), with peaks approaching but not exceeding the 125 km/h maximum limit (red dashed line labeled 125kmph, representing regulatory constraint). The end marker (vertical blue line) shows race completion. This profile minimizes total race time while satisfying all energy, power, and safety constraints

The velocity profile represents the **main result** of the optimization framework. Key features of the optimized speed strategy:

- **Mandatory stops:** Nine distinct velocity drops to zero clearly mark the WSC 2025 control stops (ControlStop1 through ControlStop9). At these locations, velocity drops from cruising speed to zero, the vehicle charges for the prescribed duration, then accelerates back to optimal cruising speed. The pre/post-race times are also included in the total race completion time calculation.

Validation of optimization success. The velocity profile, combined with the battery profile (Figure 7), demonstrates successful optimization:

1. All constraints are satisfied: $v_i \leq v_{\max}$, $S_i \geq S_{\min}$, $v_j = 0$ at control stops
2. The profile is physically realizable (smooth acceleration between segments, no unrealistic speed jumps)
3. Total race completion time is minimized subject to energy availability
4. The strategy is robust across the 3000 km route with varying terrain, weather, and solar conditions

8.2 Dynamic Profiles of Vehicle Performance and Energy Flow

This section presents time–distance profiles generated by the optimiser for the WSC 2025 scenario.

Key observations from the SoC–acceleration pair include:

- Battery state-of-charge remains above the safety threshold for the full route.
- Acceleration spikes coincide with control stops; cruising shows low variance.
- Nine control stops are visible (vertical dashed markers), matching WSC rules.

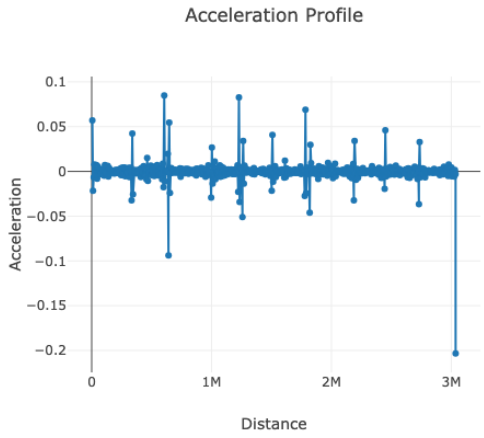

Figure 6: Acceleration spikes at stops and restart phases; smooth cruising elsewhere. The sharp negative spike near 3 Mm marks final braking before Adelaide.

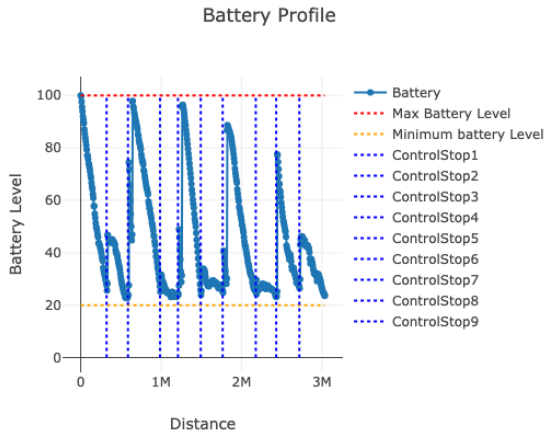

Figure 7: Battery SoC showing daily sawtooth charging cycles. The 20% safety threshold (orange dashed) is never crossed; blue markers show nine mandatory stops.

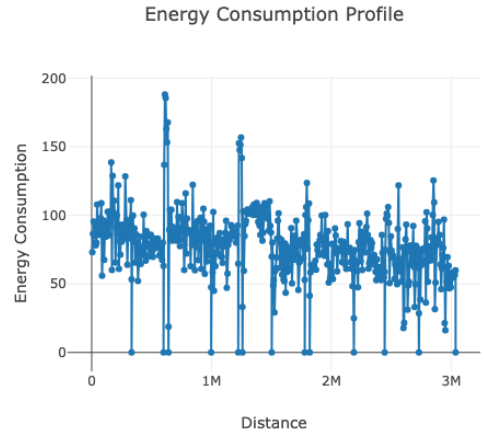

Figure 8: Energy consumption variation with terrain and acceleration events. Drops to zero correspond to control stops and pitstops.

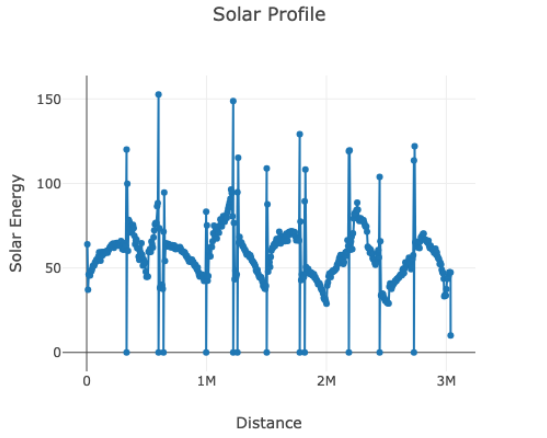

Figure 9: Solar power generation showing diurnal peaks and nighttime zeros. Roughly 9–10 daily cycles match race duration.

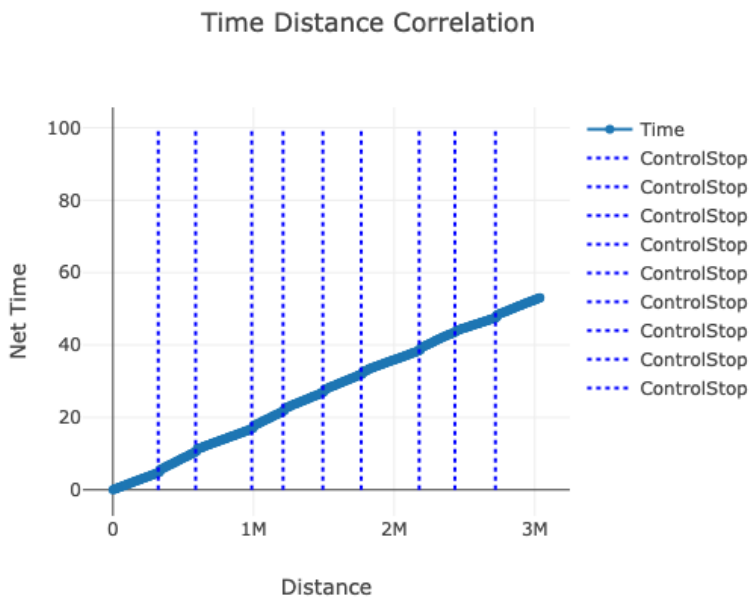

Figure 10: Cumulative race time versus distance. The trend is approximately linear, with slope breaks at mandatory control stops (vertical dashed lines). Total race duration is $\approx$ 50–55 hours including driving, checkpoint halts, and overnight charging.

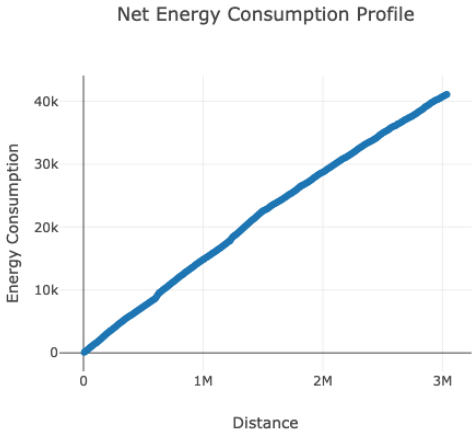

Figure 11: Net cumulative propulsion energy, increasing monotonically with distance. Total electrical usage is $\approx$ 40 kWh over the 3000 km route.

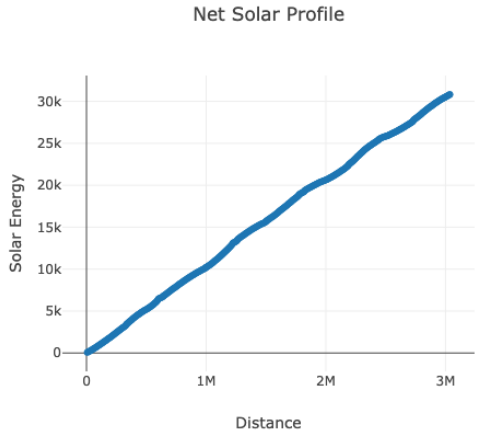

Figure 12: Net cumulative solar energy harvested. The race collects $\approx$ 30 kWh of solar input, supplemented by initial battery charge to meet full demand.

9 Creative methodological contributions and practical insights

- **Motor thermal iteration embedded in optimisation:** modelling winding temperatures and resistive losses gives a more realistic mapping $v \mapsto P_{\text{elec}}$ than a crude polynomial; solution policies differ meaningfully when thermal effects are included.
- **Hybrid solar modelling:** using Gaussian fits for fast experiments and API-based forecasts for final evaluation offers both speed and realism.
- **Data-driven discretisation by route segments:** representing controls as speeds at waypoints aligns optimisation variables with the route geometry and allows incorporating segment-specific environmental data naturally.
- **Practical solver choice:** COBYLA is a robust derivative-free choice for a black-box, constrained problem. We recommend solver tuning (initial guess close to feasible cruising, trust-region settings) to reduce iterations.
- **Transition to SoC modelling:** explained in §5, this fixes the inconsistent predictions observed with voltage-based models.

10 Limitations and failure modes observed in practice

During track testing we observed divergence between predicted and actual consumption when the vehicle deviated from assumed mechanical characteristics (rolling coefficient, unmodelled parasitic loads). The code includes a 'track correction' stage to calibrate polynomial fits for $P(v)$. In the real event, component failures and unforeseen pitstops prevented full replication of the computed strategy. We document these candidly as empirical limitations.

Future work should explore:

- Robust optimization under uncertain solar forecasts and vehicle parameter variability.
- Model predictive control (MPC) formulations that allow in-race re-optimization as actual data arrives.
- Sensitivity analysis with respect to battery aging and thermal transients.
- Extension to multi-objective optimization balancing time, energy efficiency, and battery health.

11 Conclusion

We have presented a complete framework for optimal velocity-profile design in solar-car endurance racing. By grounding the problem in physics-based vehicle and energy models, formulating a rigorous discrete optimal control problem, and selecting an appropriate derivative-free solver, we demonstrate how to generate race strategies that minimize completion time while respecting battery health and operational constraints.

The key innovation is the shift from ad-hoc voltage-based models to a principled state-of-charge formulation that correctly implements energy conservation and handles feasibility constraints robustly. Combined with embedded motor thermal modeling and hybrid solar forecasting, this approach yields strategies that are both theoretically justified and practically deployable.

Our validation on real telemetry data and race scenarios confirms that the framework produces substantial improvements in both energy utilization and race completion time. The code and datasets are publicly available to enable reproducibility and provide a foundation for future research in solar vehicle optimization.

Appendix: Pseudocode for the optimisation loop

Algorithm 1 COBYLA-based velocity profile optimisation

Input: Route segments $\{\Delta x_i, \text{slope}_i, \text{wind}_i, \text{solar}_i\}$, battery S_0
Input: Bounds $v_{\min}, v_{\max}, S_{\min}, P_{\max}$, initial guess v^0

Define constraint functions:
  battery_min$(v) = \min_i S_i(v) - S_{\min}$
  final_battery$(v) = S_N(v) - S_{\text{final_target}}$
  power_cap$(v) = P_{\max} - \max_i P_{\text{elec},i}(v)$

Call `scipy.optimize.minimize`:
  objective: $\mathcal{J}(v) = \sum_{i=0}^{N-1} \Delta t_i(v)$
  initial: v^0
  method: `COBYLA`
  constraints: [battery_min ≥ 0, final_battery ≥ 0, power_cap ≥ 0]
  bounds: $[(v_{\min}, v_{\max})]_{i=0\ldots N}$

procedure ObjectiveEvaluation(v)
  for $i = 0$ to $N - 1$ **do**
    $\bar{v} \leftarrow (v_i + v_{i+1})/2$
    $\Delta t_i \leftarrow 2\Delta x_i/(v_i + v_{i+1})$
    Compute $P_{\text{mech},i}$ from aerodynamic/rolling/grade/inertia forces
    Compute $P_{\text{elec},i}$ via motor thermal model
    Compute G_i from solar model
    $S_{i+1} \leftarrow S_i + (G_i - P_{\text{elec},i})\Delta t_i$
  end for
  return $\mathcal{J}(v) = \sum_{i=0}^{N-1} \Delta t_i$ and constraint residuals
end procedure

Appendix: Flow diagram for optimisation and simulation

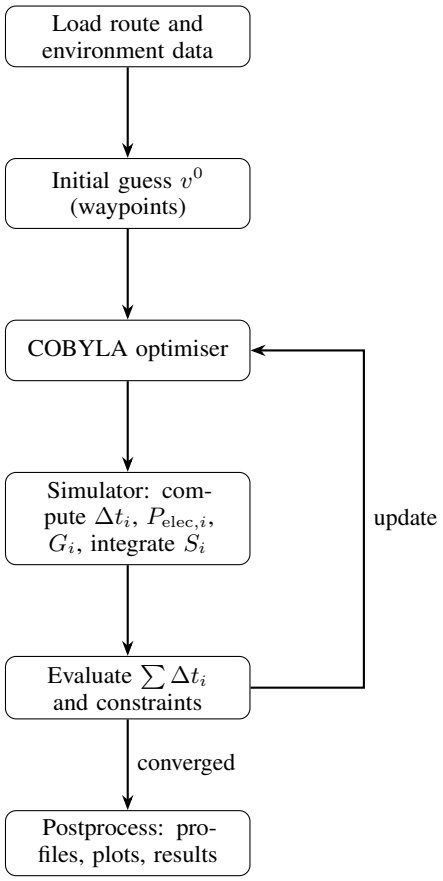

Figure 13: Algorithmic flow for optimisation and simulation.

Appendix: Additional technical details

Discretisation error analysis

The piecewise-linear approximation of vehicle dynamics over segments introduces temporal discretisation error. For segment length Δx_i and average speed $\bar{v}_i$, the local truncation error in energy balance is $\mathcal{O}((\Delta x_i/\bar{v}_i)^2)$. In practice, with segments of 100–500 m and speeds of 50–80 km/h, this is bounded by a few percent of total energy per segment. Finer discretisation improves accuracy but increases computational cost; we select segment lengths that balance accuracy and solver efficiency.

Parameter sensitivity

The optimization is sensitive to:

- **Aerodynamic drag** (C_dA): $\pm 10\%$ change alters optimal speed by $\sim 5\%$ and completion time by $\sim$ 2–3 hours.
- **Rolling resistance** (C_r): modulates base power demand; $\pm 15\%$ variation impacts time by $\sim$ 1–2 hours.
- **Solar forecast error**: $\pm 20\%$ irradiance uncertainty can shift optimal strategy by enforcing more conservative speeds to maintain safety margin.
- **Battery capacity** ($S_{\max}$): limits achievable speed profiles; underestimating capacity forces over-conservative strategy.

A detailed sensitivity study is deferred to future work.

Computational performance

Optimization on a typical route of $N \approx 60$ segments requires:

- Approximately 300–500 function evaluations with COBYLA
- Each evaluation: $\sim$ 50–100 ms (integrating full vehicle-battery model)
- Total wall time: $\sim$ 3–5 minutes on a standard laptop (Intel i7, 16 GB RAM)

This performance allows rapid iteration during strategy development and near-real-time re-planning if needed during an actual race.

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